INTEGRO-DIFFERENSIAL TENGLAMA UCHUN KOSHINING LIMITIK MASALASI YECHIMINI TADQIQ QILISH

Ne'matov Asliddin Rabbimqulovich
Jizzax Politexnika instituti assistenti

Annotatsiya.

Integral tengsizliklar hozirgi kunda integral, differensial, integro-differensial, xususiy hosilali differensial tenglamalar va boshqa ko'pgina tenglamalar yechimlarining mavjudligi, yagonaligi, boshlang'ich shartlardan uzluksiz bog'liqligi, parametrlar bo'yicha uzluksizligi va differesianllanuvchanligi, turg'unligi, chegaralanganligi va boshqa ko'pgina xossalarini o`rganishda qulay apparat bo'lib xizmat qilmoqda. Bulardan tashqari tenglamalarning aniq va taqribiy yechimlari orasidagi farqni baholashda ham tengsizliklardan foydalanish mumkin

Tayanch so'zlar.

Yadro, ozod had, echim, baholash, ketma-ket yaqinlashish usuli, integral tengsizlik, Eyler funksiyasi, Abel tipidagi yadro, Koshining limitik masalasi.

Ma'lumki Gronuoll tomonidan 1919 yilda

$$u(t) \le C + k\int_0^t u(s)ds$$

ko'rinishidagi integral tengsizlik qaralib, uning yechimi $u(t) \le Ce^{kt}$

ko'rinishida topilgan va u $y' = f(x,y), \quad y(0) = y_0$ Koshi masalasi yechimining yagonaligini isbotlashga tadbiq etilgan. Bu g'oya ko'plab chet el va O'zbekistonlik olimlarning diqqatini o'ziga tortdi va bu tengsizlik turli yo'nalishlarda umumlashtirilib mustaqil nazariyani vujudga kelishiga sabab bo'ldi. Integral tengsizliklar nazariyasi hozirgi kunda keng rivojlandi va ularga oid ma'lumotlar [1,2,3,4,5] adabiyotlarda jamlangan.

Integral tengsizliklar hozirgi kunda integral, differensial, integro-differensial, xususiy hosilali differensial tenglamalar va boshqa ko'pgina tenglamalar yechimlarining mavjudligi, yagonaligi, boshlang'ich shartlardan uzluksiz bog'liqligi, parametrlar bo'yicha uzluksizligi va differesianllanuvchanligi, turg'unligi, chegaralanganligi va boshqa ko'pgina xossalarini o`rganishda qulay apparat bo'lib xizmat qilmoqda. Bulardan tashqari tenglamalarning aniq va taqribiy yechimlari orasidagi farqni baholashda ham tengsizliklardan foydalanish mumkin [1,2,3,4,5].

Biz bu yerda yarim o'qdagi chiziqli bo'lmagan integro-differensial tenglama yechimining mavjudligi va yagonaligi haqidagi teoremani isbotlaymiz.

Ushbu

$$\frac{dx}{dt} = F(t,x) + \int_{-\infty}^{t} K(t,s,x)ds \qquad (1)$$

yarim o'qda chiziqli bo'lmagan integro-differensial tenglamani qaraymiz. Bu tenglamaning

$$\lim_{t\to-\infty} x(t) = c \qquad (2)$$

boshlang'ich shartni qanoatlantiruvchi yechimini toppish bilan shug'illanamiz. (1)-(2) birgalikda Koshining limitik masalasi deb aytiladi.

Biz avvalo (1)-(2) masalaning yechimini topishni unga ekvivalent bo'lgan integral tenglamaning yechimini toppish masalasiga keltiramiz. Haqiqatdan (1) tenglamaning (2) shartni qanoatlantiruvchi yechimi bo'lsa, u holda

$$\frac{dx(t)}{dt}=F(t,x(t))+\int_{-\infty}^{t}K(t,s,x(t))ds \qquad (3)$$

ayniyatga ega bo'lamiz. Bu tenglikning har ikkala tomonini $-\infty$ dan t gacha integrallab, (2) ni hisobga olsak ushbu integral tenglamaga kelamiz:

$$x(t)=c+\int_{-\infty}^{t}F(t,x(s))ds+\int_{-\infty}^{t}\int_{-\infty}^{s}K(s,\tau,x(\tau))d\tau ds \qquad (4)$$

Bu tenglamani ketma-ket yaqinlashish usuli yordamida yechamiz. Yechimga ketma-ket yaqinlashishlarni quyidagicha tuzamiz:

$$\begin{cases} x_0(t)=c \\ x_1(t)=c+\int_{-\infty}^{t}F(s,x_0(s))ds+\int_{-\infty}^{t}\int_{-\infty}^{s}K(s,\tau,x_0(\tau))d\tau ds \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ x_n(t)=c+\int_{-\infty}^{t}F(s,x_{n-1}(s))ds+\int_{-\infty}^{t}\int_{-\infty}^{s}K(s,\tau,x_{n-1}(\tau))d\tau ds \end{cases} \qquad (5)$$

Quyida shu $x_0(t),x_1(t),\cdots,x_n(t),\cdots$ ketma-ketlikning biror $x(t)$ funksiyaga tekis yaqinlashishini, $x(t)$ funksiyaning uzluksizligini va uning (4) tenglamani qanoatlantirishini ko'rsatamiz. (5) tengliklardagi $x_n(t)$ funksiyaning (4) tenglamaning taqribiy yechimi ham bo'ladi. Uning aniq yechim bilan farqini ham baholash mumkinligini ko'ramiz. So'ngra ma'lum shartlar bajarilganda yechimning yagonaligini ham ko'rsatamiz.

1-teorema. Faraz qilaylik

1) $F(t,x)$ funksiya $D=\{(t,x):-\infty<t<t_0,|x|<\infty\}$ sohada $K(t,s,x)$ funksiya esa $Q=\{(t,s,x):-\infty<t<t_0,-\infty<s<t,|x|<\infty\}$ sohada aniqlangan va uzluksiz funksiyalar bo'lib, bundan tashqari

$$\int_{-\infty}^{t}|F(s,c)|ds<\infty, \qquad \int_{-\infty}^{t}\int_{-\infty}^{s}|K(s,\tau,c)|d\tau ds<\infty$$

shartlar bajarilsin.

2) $F(t,x)$ va $K(t,s,x)$ funksiyalar x argumenti bo'yicha Lipshis shartini qanoatlantiradi, ya'ni

$$|F(t,x_1)-F(t,x_2)|\le\lambda(t)|x_1-x_2|$$
$$|K(t,s,x_1)-K(t,s,x_2)|\le\mu(t,s)|x_1-x_2|$$

bu yerda

$$\int_{-\infty}^{t}\lambda(t)ds<\infty, \qquad \int_{-\infty}^{t}\int_{-\infty}^{s}\mu(s,\tau)d\tau ds<\infty$$

deb hisoblanadi. U holda (3)-(4) masala $I(-\infty,t_0)$ oraliqda yagona uzluksiz yechimga ega bo'ladi. Bundan tashqari aniq va taqribiy yechimlar orasidagi farq quyidagicha baholanadi.

$$|x(t)-x_n(t)|\le\frac{a(t_0)b^{n-1}(t_0)}{n!}\exp\left\{\int_{-\infty}^{t}\left[\lambda(t)+\int_{-\infty}^{t}\mu(t,s)d\tau\right]ds\right\}$$

Isbot. Avvalo yechimning mavjudligini isbotlaymiz. Shu maqsadda (5) tenglikdan ketma-ket quyidagilarni topamiz:

$$|x_1(t)-x_0(t)|=\left|\int_{-\infty}^{t}F(s,x_0(s))ds+\int_{-\infty}^{t}\int_{-\infty}^{s}K(s,\tau,x_0(\tau))d\tau ds\right|\le\le\int_{-\infty}^{t}|F(s,c)|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}|K(s,\tau,c)|d\tau ds=a(t)$$

$$|x_2(t)-x_1(t)| = \int_{-\infty}^{t} |F(t,x_1(s))-F(t,x_0(s))|ds + \int_{-\infty}^{t}\int_{-\infty}^{s} |K(s,\tau,x_1(\tau))-K(s,\tau,x_0(\tau))|d\tau ds \le$$

$$\le \int_{-\infty}^{t} \lambda(s)|x_1(s)-x_0(s)|ds + \int_{-\infty}^{t}\int_{-\infty}^{s} \mu(s,\tau)|x_1(s)-x_0(\tau)|d\tau ds \le$$

$$\le \int_{-\infty}^{t} \lambda(s)\left[|x_1(s)-x_0(s)|ds + \int_{-\infty}^{t}\int_{-\infty}^{s} \mu(s,\tau)|x_1(\tau)-x_0(\tau)|d\tau\right]ds \le$$

$$\le a(t)\int_{-\infty}^{t} \lambda(s)\left[1+\int_{-\infty}^{s} \mu(s,\tau)d\tau\right]ds = a(t)b(t)$$

bu yerda

$$a(t) = \int_{-\infty}^{t} |F(s,c)|ds + \int_{-\infty}^{t}\int_{-\infty}^{s} |K(s,\tau,c)|d\tau ds$$

$$b(t) = \int_{-\infty}^{t} \lambda(s)\left[1+\int_{-\infty}^{s} \mu(s,\tau)d\tau\right]ds.$$

Bu jarayonni n marta takrorlasak natijada ushbu

$$|x_n(t)-x_{n-1}(t)| \le a(t)\frac{b^{n-1}(t)}{(n-1)!} \qquad (6)$$

tenglikni hosil qilamiz.

Matematik analiz kursidan ma'lumki $x_0(t), x_1(t), \cdots, x_n(t), \cdots$ ketma-ketlikning tekis yaqinlashishini ko'rsatish uchun ushbu

$$x_0(t)+(x_1(t)-x_0(t))+\cdots+(x_n(t)-x_{n-1}(t))\cdots \qquad (7)$$

qatorning tekis yaqinlashishini ko'rsatish yetarli. Biz yuqorida (7) qatorning har bir hadi uchun (6) ko'rinishdagi baholarni topgan edik. Shu sababli (7) qator uchun ushbu

$$a(t_0)+a(t_0)\frac{b(t_0)}{1!}+a(t_0)\frac{b^2(t_0)}{2!}+a(t_0)\frac{b^3(t_0)}{3!}+\cdots+a(t_0)\frac{b^{n-1}(t_0)}{(n-1)!}+\cdots$$

Qator majorant qator vazifasini bajaradi. Bu yaqinlashuvchi sonli qator bo'lganligi uchun (7) qator absolyut va tekis yaqinlashuvchi bo'ladi (Veyershtras alomatiga asosan). Demak $\{x_n(t)\}_{n=1}^{\infty}$ ketma-ketlik ham $(-\infty,t_0)$ da tekis yaqinlashadi, ya'ni

$$\lim_{n\to\infty} x_n(t) = x(t), \quad t\in(-\infty,t_0), \quad (t_0 \le \infty)$$

F va K funksiyalarning aniqlanish sohalarida uzluksiz funksiyalar bo'lganligi uchun (5) tengliklardan ko'rinadiki, $\{x_n(t)\}$ funksiyalar $(-\infty,t_0)$ oraliqda uzluksiz funksiyalarni aniqlaydi. Demak $x(t)$ funksiya ham uzluksiz va (4) tenglamani qanoatlantiradi. Haqiqatdan, $F(t,x)$ va $K(t,s,x)$ funksiyaning o'z aniqlanish sohasida uzluksiz bo'lganliklaridan (5) tengliklarning n-chisidan $n\to\infty$ da limitga o'tsak

$$\lim_{n\to\infty} x_n(t) = x(t)$$

bo'lganligi uchun,

$$x(t) = c + \int_{-\infty}^{t} F(s,x(s))ds + \int_{-\infty}^{t}\int_{-\infty}^{s} K(s,\tau,x(\tau))d\tau ds$$

tenglikka ega bo'lamiz. Bunda integral ostida limitga o'tish mumkinligi haqidagi teoremadan foydalandik. Endi yechimning yagonaligini ko'ramiz. Faraz qilaylik (4) tenglama $(-\infty,t_0)$ oraliqda $x(t)$ dan boshqa yana bir $y(t)$ yechimga ham ega bo'lsa, u holda

$$x(t)=c+\int_{-\infty}^{t}F(s,x(s))ds+\int_{-\infty}^{t}\int_{-\infty}^{s}K(s,\tau,x(\tau))d\tau ds$$

va

$$y(t)=c+\int_{-\infty}^{t}F(s,y(s))ds+\int_{-\infty}^{t}\int_{-\infty}^{s}K(s,\tau,y(\tau))d\tau ds$$

tengliklarga ega bo'lamiz. Bularning biridan ikkinchisini ayirib, hosil bo'lgan tenglikning har ikkala tomonidan modul olsak quyidagi tenglik hosil bo'ladi:

$$|x(t)-y(t)|\le\int_{-\infty}^{t}\lambda(s)|x(s)-y(s)|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}\mu(s,\tau)|x(\tau)-y(\tau)|d\tau ds$$

ixtiyoriy o'zgarmas $a\succ 0$ soni uchun quyidagi tengsizlikni yozish mumkin

$$|x(t)-y(t)|\le a+\int_{-\infty}^{t}\lambda(s)|x(s)-y(s)|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}\mu(s,\tau)|x(\tau)-y(\tau)|d\tau ds$$

Bundan quyidagiga ega bo'lamiz

$$|x(t)-y(t)|\le a\exp\left\{\int_{-\infty}^{t}\left[\lambda(s)+\int_{-\infty}^{t}\mu(s,\tau)d\tau\right]ds\right\}$$

$a\succ 0$ ning ixtiyoriyligiga ko'ra, uni nolga intiltirib quyidagi tengsizlikni hosil qilamiz

$$|x(t)-y(t)|\le 0$$

Bundan esa $x(t)-y(t)\equiv 0$ yoki $x(t)\equiv y(t)$ degan hulosaga kelamiz. Demak, (4) tenglamaning yechimi yagona ekan. Endi aniq va taqribiy yechimlar orasidagi farqlarni baholashga kirishamiz. Yuqorida qayd qilinganidek (4) tenglamaning aniq yechimi $x(t)$, taqribiy yechimi esa $x_n(t)$ bo'lsin:

$$x(t)=c+\int_{-\infty}^{t}F(s,x(s))ds+\int_{-\infty}^{t}\int_{-\infty}^{s}K(s,\tau,x(\tau))d\tau ds$$

$$x_n(t)=c+\int_{-\infty}^{t}F(s,x_{n-1}(s))ds+\int_{-\infty}^{t}\int_{-\infty}^{s}K(s,\tau,x_{n-1}(\tau))d\tau ds$$

Bularning biridan ikkinchisini ayirib hosil bo'lgan tenglikning har ikkala tomonidan modul olamiz:

$$|x(t)-x_n(t)|\le a+\int_{-\infty}^{t}\lambda(s)|x(s)-x_{n-1}(s)|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}\mu(s,\tau)|x(\tau)-x_{n-1}(\tau)|d\tau ds$$

O'ng tomondagi modullar ostidagi ifodaga $x_n(t)$ ni qo'shib va ayirib quyidagi tengsizlikni hosil qilamiz

$$|x(t)-x_n(t)|\le\int_{-\infty}^{t}\lambda(s)|x_n(s)-x_{n-1}(s)|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}\mu(s,\tau)|x_n(\tau)-x_{n-1}(\tau)|d\tau ds+$$

$$+\int_{-\infty}^{t}\lambda(s)|x(s)-x_n(s)|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}\mu(s,\tau)|x(\tau)-x_n(\tau)|d\tau ds$$

Bu tengsizlikdagi $|x_n(\tau)-x_{n-1}(s)|$ ayirma modulini (6) formula yordamida baholaymiz: u holda

$$|x(t)-x_n(t)|\le\int_{-\infty}^{t}\lambda(s)a(s)\frac{b^{n-1}(s)}{(n-1)!}+\int_{-\infty}^{t}\int_{-\infty}^{s}\mu(s,\tau)a(\tau)\frac{b^{n-1}(s)}{(n-1)!}d\tau ds+$$

$$+\int_{-\infty}^{t}\lambda(s)|x(s)-x_n(s)|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}\mu(s,\tau)|x(\tau)-x_n(t)|d\tau ds$$

yoki

$$|x(t)-x_n(t)|\le\frac{a(t)}{(n-1)!}\int_{-\infty}^{t}b^{n-1}\left[\lambda(s)\left(1+\int_{-\infty}^{t}\mu(s,\tau)d\tau\right)\right]ds+$$

$$+\int_{-\infty}^{t}\lambda(s)|x(s)-x_n(s)|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}\mu(s,\tau)|x(\tau)-x_n(\tau)d\tau|ds$$

bu yerdan quyidagiga ega bo'lamiz:

$$|x(t)-x_n(t)|\le a(t)\frac{b^n(t)}{(n)!}+\int_{-\infty}^{t}\lambda(s)|x(s)-x_n(s)|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}\mu(s,\tau)|x(\tau)-x_n(\tau)|d\tau ds$$

$a(t)\le a(t_0)$, $b(t)\le b(t_0)$ $(t\le t_0)$ ekanligini nazarga olib oxirgi tengsizlikdan integral tengsizliklar haqidagi teoremaga ko'ra quyidagiga ega bo'lamiz

$$|x(t)-x_n(t)|\le \frac{a(t_0)b^n(t_0)}{(n)!}\exp\left\{\int_{-\infty}^{t}\left[\lambda(s)+\int_{-\infty}^{s}\mu(s,\tau)d\tau\right]ds\right\}$$

bu esa izlangan bahodir. Shu bilan teorema to'liq isbotlandi.

2-teorema. Faraz qilaylik (4) tenglamada $F(t,x)$ va $K(t,s,x)$ funksiyalar quyidagi shartlarni qanoatlantirsin:

1) $F(t,x)$ funksiya

$$D=\{(t,x):-\infty<t\le t_0<0\quad |x|<\infty\}$$

sohada, $K(t,s,x)$ funksiya esa

$$Q=\{(t,s,x):-\infty<t\le t_0<0,-\infty<s\le t,\quad |x|<\infty\}$$

sohada aniqlangan va uzluksiz funksiyalar bo'lib, bundan tashqari

$$\left(\int_{-\infty}^{t}|s|^q|F(s,c)|^q ds\right)^{\frac{1}{q}}\le Mt^{-\frac{1}{q}},\ \left(\int_{-\infty}^{t}\int_{-\infty}^{s}|s|^q|\tau|^q|K(s,\tau,c)|^q d\tau ds\right)^{\frac{1}{q}}\le N\quad M>0,\ N>0$$

shartlar bajarilsin, bu yerda $\frac{1}{q}+\frac{1}{p}=1$ bo'lib, $p>1$ toq son deb hisoblaymiz.

2) $|F(t,x_1)-F(t,x_2)|\le\frac{a}{t}|x_1-x_2|,\quad a<0;\quad |K(t,s,x_1)-K(t,s,x_2)|\le\frac{b}{ts}|x_1-x_2|,\quad b>0$

U holda (4) integral tenglama yoki bari bir (1)-(2) masala yagona uzluksiz yechimga ega bo'ladi.

Isbot. (5) tengliklardan ketma-ket quyidagilarni topamiz:

$$|x_1(t)-x_0(t)|\le\int_{-\infty}^{t}|F(s,c)|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}|K(s,\tau,c)|d\tau ds\le$$

$$\le\left(\int_{-\infty}^{t}|s|^{-p}ds\right)^{\frac{1}{p}}\left(\int_{-\infty}^{t}|s|^q|F(s,c)|^q ds\right)^{\frac{1}{q}}+\left(\int_{-\infty}^{t}\int_{-\infty}^{s}|s|^{-p}|\tau|^{-p}d\tau ds\right)^{\frac{1}{p}}\left(\int_{-\infty}^{t}\int_{-\infty}^{s}|s|^q|\tau|^q|K(s,\tau,c|^q d\tau ds\right)^{\frac{1}{q}}\le$$

$$\le Mt^{-\frac{1}{q}}\left(\frac{t^{1-p}}{1-p}\right)^{\frac{1}{p}}+N\left(\frac{t^{2(1-p)}}{2(1-p)^2}\right)^{\frac{1}{p}}\le Qt^{\frac{2}{p}-2},$$

bu yerda $$Q=\frac{M+N}{\left(2(1-p)^2\right)^{\frac{1}{p}}}>0$$

Demak,

$$|x_1(t)-x_0(t)|\le Qt^{\frac{2}{p}-2}$$

Agar $\alpha=2-\frac{2}{p}\ge 1$ deb belgilab olsak, $|x_1(t)-x_2(t)|\le Qt^{-\alpha}$ tengsizlikka ega bo'lamiz.

Bunga ko'ra qiyidagilarni topamiz.

$$|x_2(t)-x_1(t)|\le\int_{-\infty}^{t}\frac{a}{s}|x_1(s)-x_0(s)|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}\frac{b}{s\tau}|x_1(\tau)-x_0(\tau)|d\tau ds\le Q\left(\frac{b}{\alpha^2}-\frac{a}{\alpha}\right)t^{-\alpha}$$

………………………………….. (8)

$$|x_n(t)-x_{n-1}(t)|\le Q\left(\frac{b}{\alpha^2}-\frac{a}{\alpha}\right)^n t^{-\alpha}.$$

Bu tengsizliklardan ko'rinadiki

$$\sum_{n=1}^{\infty}[x_n(t)-x_{n-1}(t)]$$

qator $(-\infty;t_0)$ oraliqda tekis yaqinlashadi, chunki

$$Q\sum_{n=1}^{\infty}\left(\frac{b}{\alpha^2}-\frac{a}{\alpha}\right)^n t_0^{-\alpha}\quad (t_0<0)$$

Sonli qator $\frac{b}{\alpha^2}-\frac{a}{\alpha}<1$ bo'lganda yaqinlashuvchi.

Demak, $\lim_{n\to\infty} x_n(t)=x(t)\quad(-\infty,t_0)$ oraliqdagi barcha t lar uchun tekis bajariladi. Osongina ko'rsatish mumkinki $x(t)$ funksiya (4) va demak (2)-(3) masalaning yechimi bo'ladi. Endi topilgan yechimning yagonaligini ko'rsatamiz.

Faraz qilaylik (4) tenglama ikkita $x(t)$ va $y(t)$ yechimlarga ega bo'lsin. U holda ularning har biri (4) tenglamani qanoatlantirishi kerak. Teorema shartiga ko'ra

$$|x(t)-y(t)|\le\int_{-\infty}^{t}\frac{a}{s}|x(s)-y(s)|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}\frac{b}{s\tau}|x(s)-y(s)|dsd\tau$$

Bu esa 1-teoremaga ko'ra $|x(t)-y(t)|\le 0$ yoki $x(t)\equiv y(t)$. Demak yechim yagona ekan.

Endi aniq va taqribiy yechimlar orasidagi farqni baholaymiz. $x(t)$ (1)-(2) masalaning aniq yechimi va $x_n(t)$ esa uning taqribiy yechimi (ya'ni unga $n-$chi yaqinlashish bo'lsin).

(4) va (5) larni o'zaro ayirib quyidagiga ega bo'lamiz:

$$|x(t)-x_n(t)|\le\int_{-\infty}^{t}\frac{a}{s}|x(s)-x_{n-1}(s)|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}\frac{b}{s\tau}|x(\tau)-x_{n-1}(\tau)|d\tau ds$$

O'ng tomidagi modullar orasidagi ifodaga $x_n(t)$ ni qo'shib va ayirib quyidagi tengsizlikni hosil qilamiz:

$$|x(t)-x_n(t)|\le\int_{-\infty}^{t}\frac{a}{s}|x_n(s)-x_{n-1}(s)|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}\frac{b}{s\tau}|x_n(\tau)-x_{n-1}(\tau)|d\tau ds+$$
$$+\int_{-\infty}^{t}\frac{a}{s}|x(s)-x_n(s)|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}\frac{b}{s\tau}|x(\tau)-x_{n-1}(\tau)|d\tau ds$$

Bu tengsizlikdagi $|x_n(s)-x_{n-1}(s)|$ ayirma modulini (8) formula yordamida baholaymiz

$$|x(t)-x_n(t)|\le Qa\left(\frac{b}{\alpha^2}-\frac{a}{\alpha}\right)^{n-1}\int_{-\infty}^{t}s^{-1-\alpha}ds+$$

$$+Qb\left(\frac{b}{\alpha^2}-\frac{a}{\alpha}\right)^{n-1}\int_{-\infty}^{t}\int_{-\infty}^{s} s^{-1-\alpha}\tau^{-1-\alpha}d\tau ds+$$

$$+\int_{-\infty}^{t}\frac{a}{s}\left|x(s)-x_n(s)\right|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}\frac{b}{s\tau}\left|x(\tau)-x_n(\tau)\right|d\tau ds$$

yoki

$$\left|x(t)-x_n(t)\right|\le Q\left(\frac{b}{\alpha^2}-\frac{a}{\alpha}\right)t^{-\alpha}+\int_{-\infty}^{t}\frac{a}{s}\left|x(s)-x_n(s)\right|ds+\int_{-\infty}^{t}\int_{-\infty}^{s}\frac{b}{s\tau}\left|x(s)-x_n(s)\right|d\tau ds.$$

Bu tengsizlikda $C=Q\left(\frac{b}{\alpha^2}-\frac{a}{\alpha}\right)^{n}$ $m=1$, desak

$$\left|x(t)-x_n(t)\right|\le\frac{Q\left(\frac{b}{\alpha^2}-\frac{a}{\alpha}\right)^{n}t^{-\alpha}}{1-\left(\frac{b}{\alpha^2}-\frac{a}{\alpha}\right)}$$

tengsizlik hosil bo'ladi. Shunday qilib bu ishda (1)-(2) masalaning yechimini tadqiq qilishda integral tengsizliklar haqida teorema isbotladik va uning tadbiqlarini ko'rib chiqdik.

Adabiyotlar ro'yxati

1. Неъматов, А. Р., Рахимов, Б. Ш., & Тураев, У. Я. (2016). СУЩЕСТВОВАНИЕ И ЕДИНСТВЕННОСТЬ РЕШЕНИЯ НЕЛИНЕЙНОГО УРАВНЕНИЯ ВОЛЬТЕРРА. *Ученый XXI века*, 6.
2. Нуримов Т. Филатов А.Н. Шарова Л.В. Интегральные неравенства. -Тошкент: 1990.-188 с.
3. Nurimov T. Integral tenglamalar va tengsizliklar. –Toshkent: O'qituvchi, 1985.-188 c.
4. Мамедов Я.Д. Аширов С. Нилинейные уравнения Вольтера.-Ашхабод: Ылым, 1977.-185 с.
5. Михлин С.Г. Лекции по линейным интегральным уравнениям. Физматгаз. 1959.
6. Филатов А.Н. Шарова Л.В. Интегральные неравенства и теория нелинейных колебаний. М, Наука, 1976.
7. Филатов А.Н. Методы усреднения в дифференциальных и интегро- дифференциальных уравнениях. –Тошкент: Фан, 1971.-152 с.
8. Мамедов Я.Д. Аширов С. Атдаева С. Теоремы о неравенствах. Ашхабад, "Ылым" 1980.
9. Maqsudov Sh.T. Chiziqli integral tenglamalar elementlari. Toshkent, "O'qituvchi", 1975.