MATHEMATICAL REPRESENTATION OF IMAGE PROCESSING USING DATA SEGMENTATION

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Segmentation divides an image into its constituent areas or objects. The degree of detail to which this division brought depends on the task being to solve. In other words, segmentation should be stop when objects or areas of interest detected. For example, in the task of automated control of Assembly of electronic equipment components, it is of interest to analyze images of manufactured products in order to identify certain defects, such as the absence of components or the rupture of contact tracks on the Board. Therefore, it does not make sense to perform segmentation smaller than the level of detail that is necessary to detect such defects.

Segmenting images that are not trivial is one of the most difficult image processing tasks. The ultimate success of computer image analysis procedures is largely determined by the accuracy of segmentation, for this reason, considerable attention should paid to improving its reliability. In some situations, for example, in technical control tasks, it is possible to control the shooting conditions at least to some extent. An experienced image processing system designer always pays attention to such features. In other applications, such as Autonomous target guidance systems, the developer cannot control the surrounding conditions, so the usual approach is to focus on selecting sensors of the kind that are most likely to amplify the signal from the objects of interest and at the same time reduce the influence of non-essential image details. A good example of this approach is infrared photography, which used for military purposes to detect objects with powerful thermal radiation, such as military equipment or moving troops.

Most of the image segmentation algorithms discussed in this Chapter based on one of the two basic properties of the brightness signal: discontinuity and homogeneity. In the first case, the approach is to split the image based on sharp changes in the signal, such as brightness differences in the image. The second category of methods uses splitting the image into areas that are homogeneous in the sense of pre-selected criteria. Examples of these methods include threshold processing, growing regions, merging, and splitting regions. In this Chapter, we will review and illustrate some of these approaches and show that segmentation quality improvements can achieved by combining methods from different categories, such as connecting contour selection with threshold transformation. We will also look at the morphological approach to segmentation, which is particularly attractive because it combines the positive properties of several segmentation methods described in the first part of this Chapter. In conclusion, we will consider the use of some key features that characterize the movement of objects for image segmentation.

Let denote R the entire spatial area occupied by the image. Image segmentation can considered as a process that splits R into n subdomains $R_1, R_2, \ldots, R_n$ so that

(a) $\bigcup_{i=1}^{n} \boldsymbol{R_i} = \boldsymbol{R}$,

(b) Plenty R_i is connected, i = 1, 2,..., n,

(v) $\boldsymbol{R_i} \cap \ \boldsymbol{R_j} = \emptyset$ For anyone *i* and *j*, *i* ≠ *j*,

(g) $\boldsymbol{Q}(\boldsymbol{R_i}) = \boldsymbol{TRUE}$ For *i* = 1, 2,..., *n*,

(d) $\boldsymbol{Q}(\boldsymbol{R_i} \cup \ \boldsymbol{R_j}) = \boldsymbol{FALSE}$ For anyone *related area* R_i and R_j.

Here $Q(R_k)$ is a logical predicate defined on points of the set R_k and taking a true (TRUE) or false (FALSE) value, and $\emptyset$ is an empty set. The signs $\cup$ and $\cap$ denote the operations of combining and intersecting sets, respectively. Two regions R_i and R_j are called contiguous if their Union forms a connected set. Condition (a) specifies that the segmentation must be complete, i.e. each pixel must fall into some area. Condition (b) requires that the points in the area are connected in some pre- defined sense. According to the condition (b), the regions must be disjoint. Condition (d) refers to properties that pixels in a segmented area must satisfy, for example, $Q(R_i) = TRUE$ if all R_i pixels have the same brightness. Finally, the condition (d) indicates that the two adjacent regions R_i and R_j must differ in the sense of the predicate$Q$.[1]

We see that the fundamental problem with segmentation is to divide the image into areas that meet the above conditions. Segmentation algorithms for monochrome images usually fall into one of two main categories, based on the properties of brightness values —the presence of gaps and the proximity of values. In the first category, it assumed that the edges of the regions are quite different from both the image background and from each other, which allows you to detect the border based on local brightness gaps. The prevailing approach in this category is contour — based segmentation. The second category includes area-based segmentation methods that divide an image into areas that have internal similarity according to a set of pre-defined criteria. Fig.1 illustrates the concepts introduced. For fig. 1 (a) shows an image of an area with a constant brightness on a dark background that also has a constant brightness. Together, both of these areas cover the entire image. For fig. 1 (b) the result of calculating the boundary of the inner region based on the brightness gaps is presented. Points inside and outside the border have zero values (black) because there are no brightness gaps inside the areas. To segment an image, we mark all pixels inside the border and on the border itself in one way (say, white), and all points outside the border in another way (say, black).

For fig. 1 (c) shows the result of this procedure. As you can see, it meets the conditions (a)—(b) listed at the beginning of this section. The predicate in condition (d) is as follows: if the pixel is inside or on the border, it is marked with white, otherwise it is marked with black. You can see that this predicate takes the value TRUE for the points marked in Fig. 1 (b) both black and white.

The two selected areas (object and background) also satisfy the condition (d). The following three images illustrate the segmentation based on regions. Fig. 1 (d) similar to Fig. 1 (a), but the brightness of the inner area is not constant, but forms a texture. For fig. 1 (e) shows the result of highlighting contour differences in such an image. It is clear that numerous uninformative brightness changes make it difficult to single out the border on the original image because many points with a non-zero brightness difference are connected to the true border. Therefore, the contour-based segmentation method is not suitable for this case. Note, however, that the outer region has a constant brightness, so to solve this simple segmentation problem, it is sufficient to construct a predicate that would distinguish areas with texture from areas with

[1] In General, Q can be set by a composite expression, for example, $Q(R_i) = TRUE$ if the average brightness of pixels in R_i is less than m_i, And if the standard deviation of the brightness of these pixels is greater than σ_i, where m_i and $\sigma_i$are the specified constants.

constant brightness. For this purpose, the standard deviation of the brightness values is used as a measure, since it is different from zero in the area with the texture and equal to zero in the background area. For fig. 1 (e) presents the result of splitting the original image into non-overlapping subdomains of 4×4 pixels. Each subdomain is then marked white if the standard deviation of its pixel brightness values is greater than zero (i.e., if the predicate takes a true value), or black in the opposite case. A step is visible at the border of the area, because all pixels in a 4×4 square are assigned the same brightness value. In conclusion, this result also satisfies the five conditions stated at the beginning of this section.

The derivative of a discrete function defined by the differences in its values. There are various ways to approximate derivatives by differences, but we require that any approximation of the first derivative be: (1) equal to zero in areas with constant brightness; (2) non-zero at the beginning of a step-by-step or linear change in brightness, and (3) non-zero throughout the entire section of the linear change in brightness. Similarly, an approximation of the second derivative requires that it be: (1) zero in areas with constant brightness; (2) non-zero at the beginning and end of a stepwise or linear change in brightness; and (3) zero in a section of linear change in brightness. Since we consider quantities with discrete finite values, the maximum possible change in brightness is also finite, and the shortest distance at which a change can occur is between neighboring pixels.

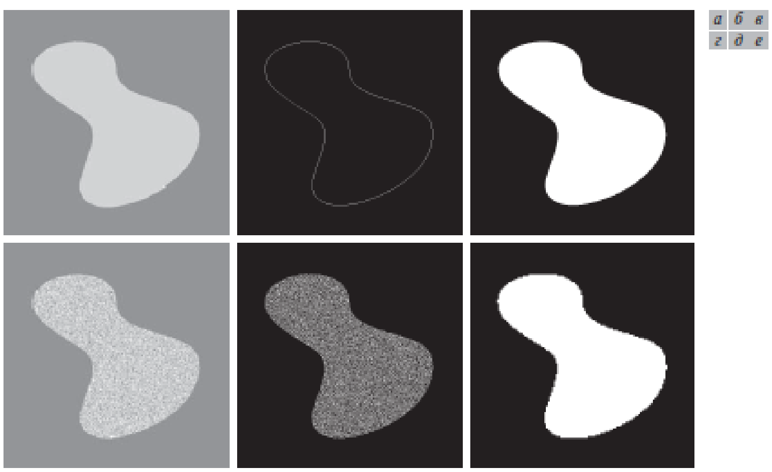

Fig. 1. (a) an Image containing an area of constant brightness. (b) the boundary of the inner region obtained from the brightness discontinuities. (c) The result of segmenting the image into two areas. (d) an Image containing an area with a texture. (e) The result of contour selection. Pay attention to the large number of contour drops within the area itself, and connected to the border of the area, which makes it difficult to single out its border only on the basis of information about the drops. (e) the Result of segmentation based on the region properties

We approximate the first-order derivative of a one-dimensional function $f(x)$ at point x by decomposing the function $f(x+\Delta x)$ into a Taylor series in the neighborhood of x, assuming $\Delta x = 1$ and leaving only linear terms. As a result, we get a discrete difference

$$\frac{\partial f}{\partial x} = f'(x) = f(x+1) - f(x) \qquad (2.1\text{-}1)$$

Here we use a partial derivative to preserve the unity of notation in the future, when we consider the image function $f(x,y)$, which depends on two variables. Then partial derivatives along the spatial axes will be used, and in the case of a one-dimensional function f, it is clear that $\partial f/\partial x = df/dx$.

Differentiating the expression (2.1-1) by x, we get the expression for the second derivative's:

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial f^2(x)}{\partial x} = f'(x+1) - f'(x) = f(x+2) - f(x+1) - f(x+1) + f(x)$$
$$= f(x+2) - 2f(x+1) + f(x),$$

where the second line follows from (2.1-1). This series expansion corresponds to the neighborhood of point $x+1$, and since we are interested in the second derivative at point x, we should subtract 1 from the value of the argument everywhere; finally, we get

$$\frac{\partial^2 f}{\partial x^2} = f''(x) = f(x+1) + f(x-1) - 2f(x) \qquad (2.1\text{-}2)$$

Consider the properties of the first and second derivatives, moving along the profile from left to right. First of all, it is clear that the first derivative is nonzero at the beginning and throughout the entire oblique difference in brightness, while the second derivative takes nonzero values only at the beginning and end of the oblique difference. Since the differences in digital images often have this form, it can be concluded that the first derivative gives off "thick" differences, and the second derivative gives off much thinner ones. Then we meet an isolated noise point. At this point, the response of the second derivative is much larger than the first. This could expected, because the second-order derivative reacts much more strongly to sudden changes than the first derivative. Thus, the second derivative tends to amplify small parts (including noise) to a much greater extent than the first-order derivative. The line in this example is relatively thin and therefore also a "fine detail", so we again see that the value of the second derivative on it is much higher.

In conclusion, we note that on both inclined and step differences the second derivative has two bursts of opposite signs (moving from negative to positive or from positive to negative at the beginning and at the end of the difference). This effect of "doubling the difference" is an important characteristic that can be used to find differences. The sign of the second derivative also allows you to determine whether the point is the beginning of the change from light to dark (negative second derivative) or from dark to light (positive second derivative), and the sign appears at the beginning (and end) of the edge.

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

Fig. 2 A general representation of a spatial filter mask with a size of 3×3 elements

As a result, we come to the following conclusions.

(1) First-order derivatives generally distinguish wider differences in the image.

(2) Second-order derivatives give a stronger response to small details such as thin lines, isolated dots, and noise.

(3) The second derivatives at oblique and stepwise differences in brightness give a response twice.

(4) The sign of the second derivative can be used to determine the direction of the difference in brightness (from light to dark or vice versa).

The method of calculating the first and second derivatives in each pixel of the image is to use spatial filters. For the one shown in fig. 2 filter masks with dimensions of 3 × 3 elements. This procedure consists in calculating a linear combination of mask coefficients with brightness values of image elements covered by the mask. When using this mask, the response at the image point coinciding with the center of the mask is given by

$$R = w_1 z_1 + w_2 z_2 + \ldots + w_9 z_9 = \sum_{k=1}^{9} w_k z_k \qquad (2.1\text{-}3)$$

where z_k— pixel brightness value corresponding to the coefficient w_k masks. Derivatives calculated by spatial filtering of the image using spatial masks, as mentioned in the sections above.

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