ORTHONORMAL WAVELETS IN THE HAAR TRANSFORM

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The third and last method used in image processing, which is related to multiple-scale theory, is the Haar transform. In the context of this Chapter, the significance of this transformation is due to the fact that its basic functions (defined below) form the first and simplest known system of orthonormal wavelets. These wavelets will be used in a number of examples later.

The Haar transform is separable and symmetric and according to the discussion is written in matrix form as follows:

$$T = HFH^T \qquad (1.3.1)$$

where F is the image matrix, H is the transformation matrix, and T is the result of the transformation (each with dimensions N×N). The Superscript T indicates a matrix transposition operation; transposition is necessary because the matrix H is not symmetric.

a,b,c,d

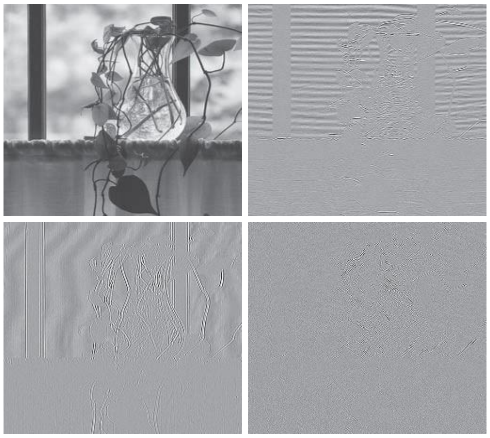

Fig.1. Decomposing the image with the vase in figure 6 using the sub-band encoding system (in figure 6) into four sub-bands: (a) approximations, (b) horizontal details, (C) vertical details, (d) diagonal details

The Haar transformation matrix H consists of Haar basis functions hk(z). These functions are defined on a continuous closed interval z ∈ [0,1] for k = 0, 1, 2,..., N-1, where N = 2n. To get H, we set an integer k such that k = 2p + q-1, where 0 ≤ p ≤ n – 1, q = 0 or 1 for p = 0 and 1 ≤ q ≤ 2p when p ≠ 0. then the Haar basis functions will be

$$h_0(z) = h_{00}(z) = \frac{1}{\sqrt{N}}, \quad z \in [0,1] \qquad (1.3\text{-}2)$$

$$h_k(z) = h_{pq}(z) = \frac{2^{p/2}}{\sqrt{N}} \begin{cases} 1 \quad by\ (q-1)/2^p \le z < (q-0{,}5)/2^p; \\ -1 \ \ by\ (q-0{,}5)/2^p \le z < (q)/2^p; \\ \quad 0 \quad in\ other\ cases,\ z \in [0,1] \end{cases} \qquad (1.3\text{-}3)$$

The row with the number i of the Haar transformation matrix H with dimensions N×N consists of the values of the function hi (z) taken at the points z = 0/N, 1/N, 2/N,..., (N-1)/N. For

N = 2, for example, the first row of the 2×2 Haar matrix is calculated as h0(z) for z = 0/2, 1/2. According to equation (7.1-16), h0(z) is equal to 1/2 regardless of z, so that the first row of H2 contains two identical elements 1/2. The second line is obtained by calculating h1(z) for z = 0/2, 1/2. Since k = 2p + q-1, for k = 1 we have: p = 0 and q = 1. Thus, from equation (7.1-17) we get: h1(0) = 20/ 2= 1/ 2 and h1(1/2) = -20/ 2= -1/ 2. Thus, the 2×2 matrix of the Haar transformation is equal to:

If N = 4, the indexes k, q, and p take values

k	**p**	**q**
0	0	0
1	0	1
2	1	1
3	1	2

and the 4×4 transformation matrix H4 has the form

$$H_4 = \frac{1}{\sqrt{4}}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ \sqrt{2} & \sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{2} & \sqrt{2} \end{bmatrix} \qquad (1.3\text{- }4)$$

The Haar transform is of interest mainly because the rows of the H2 matrix can be used to form the analysis filters h0(n) and h1(n) of the two-element Bank of exact recovery filters (see the previous section), as well as refinement sequences for the scaling function and wavelets of the simplest and oldest wavelet transform.

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